The expansion waves travel more rapidly than the compression waves, because the local gas temperature and, hence, the local speed of sound are higher. The higher gas temperatures are the result of the preceding compression processes described in conjunction with Fig. 4.

These results were analyzed by constructing both wave diagrams and state-plane diagrams. Excellent agreement was obtained between the calculated and measured values for various gas dynamic states in a resonance cycle.

Conclusions

The experimental investigation revealed an excellent correlation between the transient pressures and time-averaged temperatures measured at the endwall of a blunt, axisymmetric resonance tube. During various portions of a resonance cycle, transient pressures in the tube exceeded the jet stagnation pressure and, at other times, fell below the ambient pressure level. The reported transient wave pressure amplitudes and the wave speed data can be used to construct wave diagrams and state-plane diagrams. These diagrams are extremely useful in the analysis of the resonance tube phenomenon.

Acknowledgment

This research was carried out with support from the Rutgers Research Council.

References

[1] Skok, M. W. and Page, R. H., "An Analog Investigation of the Gas Jet Resonance Tube," 5th Cranfield Fluidics Conference, Paper No. D3-37, 1972.

[2] Przirembel, C. E. G., Page, R. H. and Wolf, D. E., "Visual Studies of Resonance Tube Phenomena," *Proceedings of the International Symposium on Flow Visualization*, Tokyo, Japan, Oct. 1977, pp. 205-210.

[3] Brocher, E., Maresca, C. and Bournay, M. H., "Fluid Dynamics of the Resonance Tube," *Journal of Fluid Mechanics*, Vol. 43, Pt. 2, 1970, pp. 369-384.

[4] Wu, J. H. T., Neemeh, R. A. and Ostrowski, P. P., "Resonance Tubes in Under-expanded Jets," *Transactions, Canadian Aeronautics and Space Institute*, Vol. 6, No. 1, 1973, pp. 26-35.

[5] Wu, J. H. T., Neemeh, R. A. and Ostrowski, P. P., "Subsonic Jet Driven Resonance Tube," *Transactions, Canadian Aeronautics and Space Institute*, Vol. 8, No. 1, 1975, pp. 29-34.

J80-~~219~~ 218

Steady Oblique Shock-Wave Reflections in Perfect and Imperfect Monatomic and Diatomic Gases

20015

G. Ben-Dor*
Ben-Gurion University of the Negev, Beer-Sheva, Israel

I. Introduction

THE reflection of oblique shock-waves in steady and pseudosteady flows is a nonlinear problem which has been investigated by many researchers for more than three decades. However, only recently, have the oblique shock-wave equations that describe the two and three shock confluences also been solved for imperfect gases.[1-3] Utilizing this solution, the domains and transition boundaries of different types of reflection were established in diatomic[2] and monatomic[3] gases. The analysis was substantiated experimentally. Consequently, it is now possible to predict the type of reflection for a given set of initial conditions.

During the study of the reflection phenomena, it was realized that the domains and transition boundaries of oblique shock-wave reflections in steady flow are not defined completely.

Consequently, it was decided to extend this work to steady flows. This, it is believed will undoubtedly bring added light and order into this complex problem.

II. Analysis

In the following, the different types of oblique shock-wave reflections possible in steady flows, as well as the transition criteria between them, are discussed.

Four types of oblique shock-wave reflections are possible in pseudosteady flows: 1) regular reflections (RR), 2) single-Mach reflection (SMR), 3) complex-Mach reflection (CMR), and 4) double-Mach reflection (DMR). In steady flows, however, only RR and SMR are possible.

Three different criteria for the transition RR $\rightleftharpoons$ SMR exist in the literature. The most quoted criterion is that due to von Neumann,[4] which is based on the fact that in RR the deflection of the flow by the reflected shock wave R is equal in magnitude, but opposite in sign, to the deflection by the incident shock wave I; therefore, $\theta_1 + \theta_2 = 0$. This is violated when θ_1 exceeds in magnitude the maximum deflection angle θ_{2m}. This criterion, referred to as the "detachment" criterion (the term detachment comes from steady flows where the oblique shock-wave detaches at this angle), has the following form:

$$\theta_1 + \theta_{2m} = 0 \tag{1}$$

Because of the disagreement between Eq. (1) and steady flow experiments, Henderson and Lozzi[5] introduced an alternative criterion based on the observation that, in order to maintain the system in mechanical-equilibrium, the RR $\rightleftharpoons$ SMR transition should take place at the point where the R-polar intersects the I-polar (SMR solution) on the P/P_0-axis (RR solution). Consequently, the mechanical-equilibrium criterion implies that

$$\theta_1 + \theta_2 = \theta_3 = 0 \tag{2}$$

It should be noted that the condition given by Eq. (2) is not always possible. Kawamura and Saito[6] showed that for incident flow Mach numbers M_0 smaller than a certain "change over" value M_{0c}, the R-polar becomes tangent to the P/P_0-axis, inside the I-polar, and the situation described by Eq. (2) is unobtainable. For a diatomic gas, $M_{0c} = 2.20 \pm 0.03$, while for a monatomic gas $M_{0c} = 2.46 \pm 0.01$.

Hornung et al.[7] initiated another criterion for the termination of RR. They argued that in order for a SMR to form, a length scale must be available at the reflection point, i.e., pressure signals must be communicated to the reflection point. Unlike the "mechanical-equilibrium" criterion, the "length scale" criterion applies to the entire range of M_0. However, while for $M_0 \geq M_{0c}$ it is described by Eq. (2), for $M_0 < M_{0c}$ it reduces to the "sonic" criterion, i.e., RR $\rightleftharpoons$ SMR transition occurs when the flow behind the reflected shock-wave R becomes subsonic. Analytically, the sonic criterion can assume one of the following forms:

$$\theta_1 + \theta_{2s} = 0 \quad \text{or} \quad M_2 = 1 \tag{3}$$

It should be mentioned that in practice the boundary lines obtained from Eqs. (1) and (3) are too close to the resolved experimentally. Note also that the general criterion of Hornung et al.[7] results in two different transition lines for

Received March 14, 1979; revision received Jan. 16, 1980.

Index category: Shock Waves and Detonations.
*Lecturer, Dept. of Mechanical Engineering. Member AIAA.

steady and pseudosteady flows. In steady flows, Eq. (3) holds for $M_0 < M_{0c}$ and Eq. (2) for $M_0 \geq M_{0c}$. In pseudosteady flows, however, Eq. (3) or (1) is the transition criterion for the entire range of M_0.

Domains of Different Types of Reflections in (M_0, ϕ_0) Plane

The parameters M_0 and ϕ_0 are traditionally[4,5] used to describe the domains of different types of reflection in steady flows since they are directly related to the parameters used in the pseudosteady analysis,[1-3] i.e., the incident shock Mach number $M_s = M_0 \sin\phi_0$ and the effective wedge angle $\theta_w' = 90 - \phi_0$. The value of θ_w' equals θ_w (actual wedge angle) for RR and $\theta_w + \chi$ (χ is the triple point trajectory angle) for SMR, CMR, and DMR.

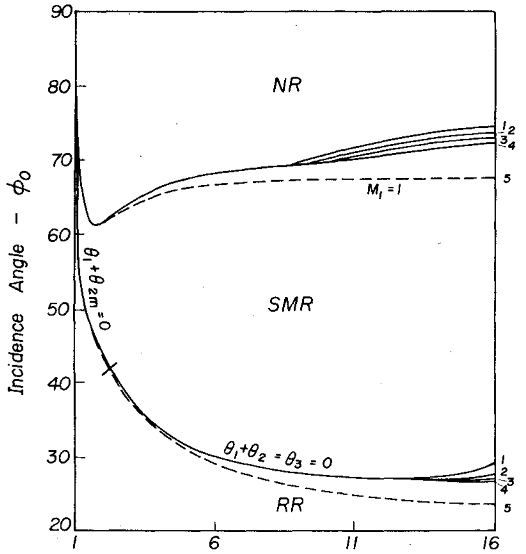

Fig. 1 Domains and boundaries of oblique shock-wave reflections in steady flows. Lines 1-4 represent imperfect nitrogen with $P_0 = 1$, 10, 100, and 1000 Torr, respectively, and $T_0 = 300$ K, line 5 represents perfect diatomic gas with $\gamma = 7/5$.

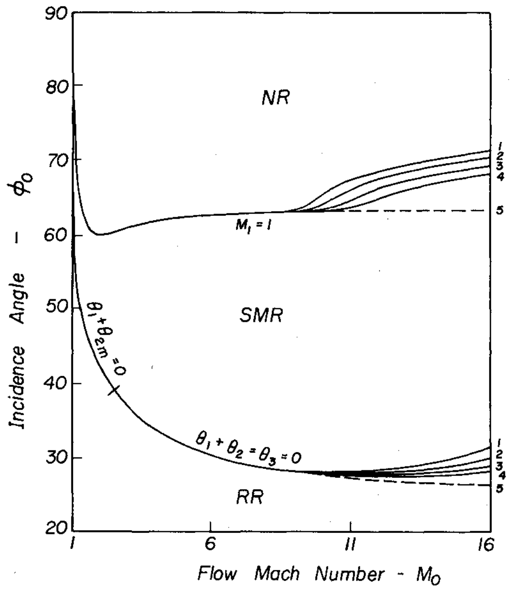

Fig. 2 Domains and boundaries of oblique shock-wave reflections in steady flows. Lines 1-4 represent imperfect argon with $P_0 = 1$, 10, 100, and 1000 Torr, respectively, and $T_0 = 300$ K, line 5 represents perfect monatomic gas with $\gamma = 5/3$.

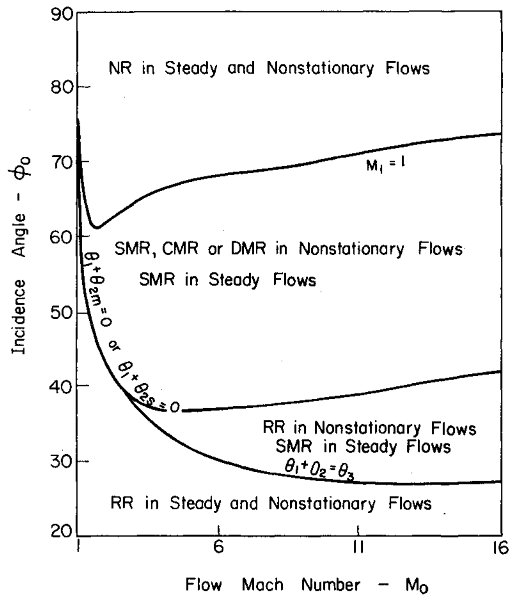

Fig. 3 Comparison between steady and pseudosteady domains of different types of reflection in the (M_0, ϕ_0) plane. Note the large reduction in the RR-domain of steady flows due to the different RR-termination line, nitrogen, $P_0 = 15$ Torr, $T_0 = 300$ K.

The domains of different oblique types of reflection in nitrogen are shown in Fig. 1. The solid boundary lines are for imperfect nitrogen in dissociation equilibrium, while the dashed lines are for a perfect diatomic gas with $\gamma = 7/5$. The (M_0,ϕ_0) plane is divided into three domains of RR, SMR, and NR (no reflection). The NR domain corresponds to the unobtainable strong solution, i.e., $M_I < 1$. The significance of real gas effects on shifting the boundary lines is clearly seen. The perfect gas theory can be used only in the narrow range, i.e., $1 < M_0 < 2.22$. At $M_0 = 2.22$, the imperfect gas lines start to diverge from the perfect gas line due to vibrational excitation. At high Mach numbers ($M_0 > 8.60$), the imperfect gas line splits into a multiplicity of lines for different initial pressures ($P_0 = 1$, 10, 100, and 1000 Torr) at a constant initial temperature ($T_0 = 300$ K) due to dissociation. The lower the initial pressure, the greater the divergence of the real boundary line from the perfect gas line. At even higher Mach numbers electronic excitation and ionization would play a similar role. When M_0 approaches unity the line $\theta_I + \theta_{2s} = 0$ approaches the line $M_I = 1$. These two lines coincide at $M_0 = 1$ and $\phi_0 = 90$ deg.

The domains of RR, SMR, and NR for argon are shown in Fig. 2. The solid lines are for imperfect argon in ionization equilibrium while the dashed lines are for a perfect monatomic gas with $\gamma = 5/3$. Unlike the foregoing case of nitrogen, where the perfect gas theory was adequate only for $M_0 < 2.22$ (Fig. 1), the perfect gas model can be used in argon over a much wider range, i.e., $1 < M_0 < 8.95$. At $M_0 = 8.95$, the imperfect gas boundary lines start to split due to electronic excitation and ionization. As expected, the shift depends on the initial pressure (at a constant initital temperature). The lower the initial pressure, the greater the shift of the corresponding boundary line. Consequently, one can expect a SMR when $M_0 = 16$, $\phi_0 = 30$ deg, and $P_0 \geq 10$ Torr, while a RR will be obtained if $P_0 \leq 1$ Torr.

It was shown earlier that the criterion for the termination of RR that was recently developed by Hornung et al.[7] implies that the RR termination line depends upon whether the flow is steady or pseudosteady. The difference between these two transition lines can best be seen in Fig. 3, where both lines are drawn. It is seen that a large area exists in which the kind of reflection depends on the type of flow. In pseudosteady flows, an RR will be obtained, while in steady flows a SMR, CMR, or DMR is expected in that region.

III. Conclusion

The present work on the reflection of oblique shock-waves in steady flows compliments the previous work on the pseudo-steady oblique shock-wave reflection.[2,3] Similarly, it establishes the domains of different types of reflection and their transition boundaries, and reveals the significance of real gas effect on shifting the boundary lines between the various domains. The domains and their boundaries are established for both perfect and imperfect monatomic (argon) and diatomic (nitrogen) gases. The present work for steady flows, as well as that already reported for pseudosteady flows, has brought new insight and order into these complex problems.

Acknowledgments

The author wishes to thank I. I. Glass of UTIAS for his advice and invaluable help during the course of this investigation. The financial assistance from the National Research Council of Canada and the U.S. Air Force under Contract AF-AFOSR 77-3303 is gratefully acknowledged.

References

[1] Ben-Dor, G., "Regions and Transitions of Non-Stationary Oblique Shock-Wave Diffractions in Perfect and Imperfect Gases," UTIAS Rept. No. 232, Aug. 1978.

[2] Ben-Dor, G. and Glass, I. I., "Domains and Boundaries of Non-Stationary Oblique Shock-Wave Reflections: I. Diatomic Gas," *Journal of Fluid Mechanics*, Vol. 92, Pt. 3, June 1979, pp. 459-496.

[3] Ben-Dor, G. and Glass, I. I., "Domains and Boundaries of Non-Stationary Oblique Shock-Wave Reflections: II. Monatomic Gas," *Journal of Fluid Mechanics*, Vol. 96, Pt.4, Feb. 1980, pp. 735-756

[4] Neumann von, J., *Collective Works*, Vol. 6, Pergamon Press, New York, 1963.

[5] Henderson, L. F. and Lozzi, A., "Experiments on Transition of Mach Reflection," *Journal of Fluid Mechanics*, Vol. 68, Pt. 1, March 1975, pp. 139-155.

[6] Kawamura, R. and Saito, H., "Reflection of Shock-Waves-1 Pseudo-Stationary Case," *Journal of the Physics Society of Japan*, Vol. 11, No. 5, May 1956, pp. 584-592.

[7] Hornung, H. G., Oertel, Jr., H., and Sandenan, R. J., "Transition to Mach Reflection of Shock-Waves in Steady and Pseudo-Steady Flows with and without Relaxation," *Journal of Fluid Mechanics*, Vol. 90, Pt. 3, 1979, pp. 541-560.

J80-219

Response of Duffing Oscillator to One Half-Cycle Sine Pulse

00033

H. R. Srirangarajan* 20001
Aeronautical Development Establishment, 30003
Bangalore, India

Nomenclature

A	= amplitude
f	= transformation function
F	$= P/m$
$J_\lambda(n\pi)$	= Bessel function of the first kind
K	= linear spring stiffness
m	= mass
n	= constant
p	$=\sqrt{K/m}$ = natural frequency
P	= magnitude of force
t	= time
T	= natural period
X_S	$=P/K$ = static displacement
x,y	= displacements
ϵ	$=\mu/m$ = nonlinear parameter
$\bar{\epsilon}$	$=\epsilon x_s^2/\omega^2$
$\Gamma(\ )$	= gamma function
λ	= ultraspherical polynomial index
μ	= nonlinear spring stiffness
ω	= forcing frequency
τ	= pulse period
θ	= phase
$(\dot{\ })$	= differentiation with respect to time

Received July 25, 1979; revision received Feb. 15, 1980.
Index categories: Aeroacoustics; Analytical and Numerical Methods; Vibration.
*Senior Scientific Officer, Structures and Materials Division.

Introduction

THE method of ultraspherical polynomial approximation has been extensively used to solve many nonlinear problems.[1-3] Here this method is utilized to obtain the response of the Duffing oscillator subjected to one half-cycle sine pulse. The corresponding linear problem is solved by Jacobsen.[4] It is shown that the method gives accurate results for nonresonant solutions, whereas it fails to give even qualitatively accurate solutions for resonant solutions. An example is solved and time-displacement solutions are obtained for many values of the ratio of pulse period to natural period. First peak and time taken for first peak are plotted vs τ/T.

Method

Consider the Duffing equation

$$m\ddot{x}+Kx+\mu x^3=P\sin\omega t \quad \text{for } t<\tau \tag{1}$$

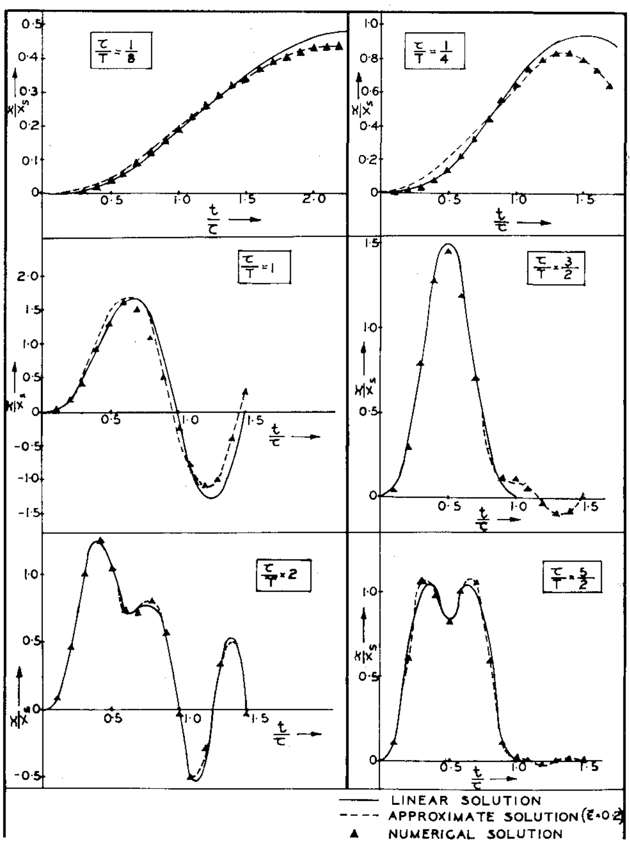

Fig. 1 Time-displacement curves for nonresonance solutions for various values of τ/T.